

Find $\lim_{x \rightarrow \infty} \coth x$ algebraically.

SCORE: _____ / 3 PTS

$$\lim_{x \rightarrow \infty} \frac{e^{2x} + 1}{e^{2x} - 1} = \lim_{x \rightarrow \infty} \frac{2e^{2x}}{2e^{2x}} = 1$$

① $\frac{\infty}{\infty}$ L'HÔPITAL'S RULE ①

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook.

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NOTE: This is NOT a question about derivatives.

$$y = \sinh^{-1} x$$

$$\textcircled{1} \quad x = \sinh y = \frac{e^y - e^{-y}}{2}$$

$$\textcircled{\frac{1}{2}} \quad x = \frac{z - \frac{1}{z}}{2}$$

$$2x = z - \frac{1}{z}$$

$$2xz = z^2 - 1$$

$$\textcircled{\frac{1}{2}} \quad 0 = z^2 - 2xz - 1$$

$$\textcircled{1} \quad z = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = x \pm \sqrt{x^2 + 1} \quad \textcircled{\frac{1}{2}}$$

$$\text{LET } z = e^y, \text{ so } z > 0 \quad \textcircled{\frac{1}{2}}$$



$$e^y = z = x + \sqrt{x^2 + 1} \quad \textcircled{\frac{1}{2}}$$

$$\sinh^{-1} x = y = \ln(x + \sqrt{x^2 + 1}) \quad \textcircled{\frac{1}{2}}$$

If $\coth x = -7$, find $\sinh x$.

ALTERNATE SOLUTION ON

SCORE: ____ / 4 PTS

$$\cosh^2 x - \sinh^2 x = 1$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x \quad (1)$$

$$48 = \operatorname{csch}^2 x \quad (1)$$

$$\operatorname{csch} x = \pm 4\sqrt{3} \rightarrow$$
$$= -4\sqrt{3} \quad (\pm 2)$$

$$\sinh x = -\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12} \quad (\pm 2)$$

$$\coth x = \frac{\cosh x}{\sinh x} \quad \text{NEXT PAGE}$$

SINCE $\coth x < 0$
AND $\cosh x > 0$ (FOR ALL x),
SO $\sinh x < 0$ (± 2)
AND $\operatorname{csch} x < 0$

If $\coth x = -7$, find $\sinh x$.

$$\tanh x = -\frac{1}{7}$$

① POINT
EACH

$$\frac{\sinh x}{\cosh x} = -\frac{1}{7}$$

$$\sinh x = -\frac{1}{7} \cosh x$$
$$= -\frac{1}{4\sqrt{3}} = -\frac{\sqrt{3}}{12}$$

SCORE: _____ / 4 PTS

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\frac{48}{49} = \operatorname{sech}^2 x$$

$$\frac{4\sqrt{3}}{7} = \operatorname{sech} x > 0$$

$$\frac{7}{4\sqrt{3}} = \cosh x$$

Find $\frac{d}{dx} x^5 \cosh^{-1}(x^3)$. Simplify your final answer as a single fraction.

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You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$5x^4 \cosh^{-1} x^3 + x^5 \frac{3x^2}{\sqrt{(x^3)^2 - 1}}$$

$$\underbrace{5x^4 \cosh^{-1} x^3}_{(1)} + \underbrace{\frac{3x^7}{\sqrt{x^6 - 1}}}_{(2)}$$

For this question, you may use the formulae for $\frac{d}{dx} \sinh x$, $\frac{d}{dx} \cosh x$ and/or $\frac{d}{dx} \tanh x$ without proving them. SCORE: _____ / 8 PTS

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] Without using the logarithmic formula for $\tanh^{-1} x$, prove the formula for $\frac{d}{dx} \tanh^{-1} x$.

$$y = \tanh^{-1} x$$

$$\textcircled{1} \quad x = \tanh y$$

$$\textcircled{1\frac{1}{2}} \quad 1 = \operatorname{sech}^2 y \frac{dy}{dx}$$

$$1 = (1 - \tanh^2 y) \frac{dy}{dx}$$

$$1 = (1 - x^2) \frac{dy}{dx}$$

$$\textcircled{1\frac{1}{2}} \quad \frac{dy}{dx} = \frac{1}{1 - x^2}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\textcircled{1} \quad 1 - \tanh^2 y = \operatorname{sech}^2 y$$

[b] Without using the exponential formula for $\operatorname{sech} x$, prove the formula for $\frac{d}{dx} \operatorname{sech} x$.

$$\frac{d}{dx} \frac{1}{\cosh x} = - (\cosh x)^{-2} \sinh x \quad \textcircled{2}$$

$$= - \frac{1}{\cosh x} \frac{\sinh x}{\cosh x}$$

$$= - \operatorname{sech} x \tanh x \quad \textcircled{1}$$